Flatté-like distributions and the $a_0(980)/f_0(980)$ mesons.

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Introduction

$$\frac{d\sigma_i}{dm} \sim \left| \frac{m_R \cdot \Gamma_i}{m_R^2 - m^2 - i\, m_R (\Gamma_{\pi\eta} + \Gamma_{K\bar{K}})} \right|^2$$

$i = \pi\eta$ or $K\bar{K}$ $\quad R = a_0$

$m_R \simeq 2 m_K$ $\quad \Gamma_{K\bar{K}} = \begin{cases} \frac{\bar{g}_K}{2}\sqrt{m^2 - 4m_K^2} & \text{above thr.} \\ i\,\frac{\bar{g}_K}{2}\sqrt{4m_K^2 - m^2} & \text{below thr.} \end{cases}$

3 free parameters: $\bar{g}_\eta (\Gamma_{\pi\eta})$, $\bar{g}_K$ and m_R

- The uncertainties in the parameters for the f_0 and a_0 mesons remain large!

a_0

Ref.	m_R	$\bar{g}_\eta$	$\bar{g}_K$	$R = \frac{\bar{g}_K}{\bar{g}_\pi}$	α
Teige	1001	0.218	0.224	1.03	0:276
Bugg	999	0.454	0.516	1.14	0.105
Abele	999	0.215	0.222	1.03	0:221
KLOE[a]	984.8	0.376	0.412	1.1	-0.28
N.Achasov[a]	992	0.453	0.56	1.24	0.006
SND[a]	995	0.389	1.414	3.63	0.027

f_0

Ref.	m_R	$\bar{g}_\pi$	$\bar{g}_K$	R	α
SND[a]	969.8	0.417	2.51	6.02	-1.35
CMD2[a]	975	0.317	1.51	4.76	-1.00
KLOE[a]	973	0.538	2.84	5.28	-1.07
OPAL	957	0.09	0.97	10.78	-1.60
E791	977	0.09	0.02	0.22	-0.66

Scaling behavior

$$f_p = -\frac{1}{2\,q_p^{th}}\;\frac{\Gamma_p}{E - E_{BW} + i\frac{\Gamma_p}{2} + i\frac{\bar{g}_K}{2}\,k}$$

$f_p \to \pi\pi,\ \pi\eta$

$$\Gamma_p = \bar{g}_p \cdot q_p^{th} \qquad E_{BW} = m_R - 2m_K$$

$$E = \sqrt{s} - 2m_K \qquad k = \begin{cases} \sqrt{m_K E} & E > 0 \\ \sqrt{-m_K E} & E < 0 \end{cases}$$

$$f_p = \frac{\bar{g}_p}{\bar{g}_K}\;\frac{1}{-\frac{1}{a_{K\bar{K}}} + \frac{r_{K\bar{K}}}{2}k^2 - ik}$$

$$a_{K\bar{K}} = -\frac{\bar{g}_K}{2\left(E_{BW} - i\frac{\Gamma_p}{2}\right)} \qquad r_{K\bar{K}} = -\frac{4}{m_K\,\bar{g}_K}$$

Scale transformation

$$E_{BW} \to \lambda E_{BW} \qquad \Gamma_p \to \lambda \Gamma_p \qquad \bar{g}_K \to \lambda \bar{g}_K$$

$$\Downarrow$$

$$a_{K\bar{K}} \xrightarrow{\lambda} a_{K\bar{K}} \qquad r_{K\bar{K}} \xrightarrow{\lambda} \frac{r_{K\bar{K}}}{\lambda}$$

Effective range approximation is not scale invariant

$$f_p = \frac{\bar{g}_p}{\bar{g}_K} \frac{1}{-\frac{1}{a_{K\bar{K}}} + \frac{1}{\lambda} \frac{r_{K\bar{K}}}{2} \kappa^2 - i\kappa}$$

Scattering length approximation is scale invariant

$$f_p = \frac{\bar{g}_p}{\bar{g}_K} \frac{1}{-\frac{1}{a_{K\bar{K}}} - i\kappa}$$

$$a_{K\bar{K}} = \frac{-\bar{g}_K}{2(E_{BW} - \frac{i}{2}\Gamma_p)}$$

$$R = \frac{\bar{g}_K}{\bar{g}_p} \qquad \alpha = \frac{2E_{BW}}{\Gamma_p}$$

$$\Downarrow$$

$$f_p = \frac{1}{\alpha q_p^{th} - i(q_p^{th} + R\kappa)}$$

q_p^{th} from PDG

$$\sigma_p = \frac{4\pi}{(q_p^{th})^2} \frac{1}{1+\alpha^2} \begin{cases} 1 - \frac{2R}{1+\alpha^2} \frac{\kappa}{q_p^{th}} & E > 0 \\ 1 - \frac{2R\alpha}{1+\alpha^2} \frac{\kappa}{q_p^{th}} & E < 0 \end{cases}$$

Figure 1: $f_{\pi\pi} = \frac{\bar{g}_\pi}{\bar{g}_K} \frac{1}{\frac{-1}{a_{K\bar{K}}} + \frac{1}{\lambda}\frac{r_{K\bar{K}}}{2}k^2 - ik}$; $\sigma_{\pi\pi} = 4\pi|f_{\pi\pi}|^2$,

$\bar{g}_\pi = 0.317,\ \bar{g}_K = 1.51,\ |E_r| = 16.3 MeV$

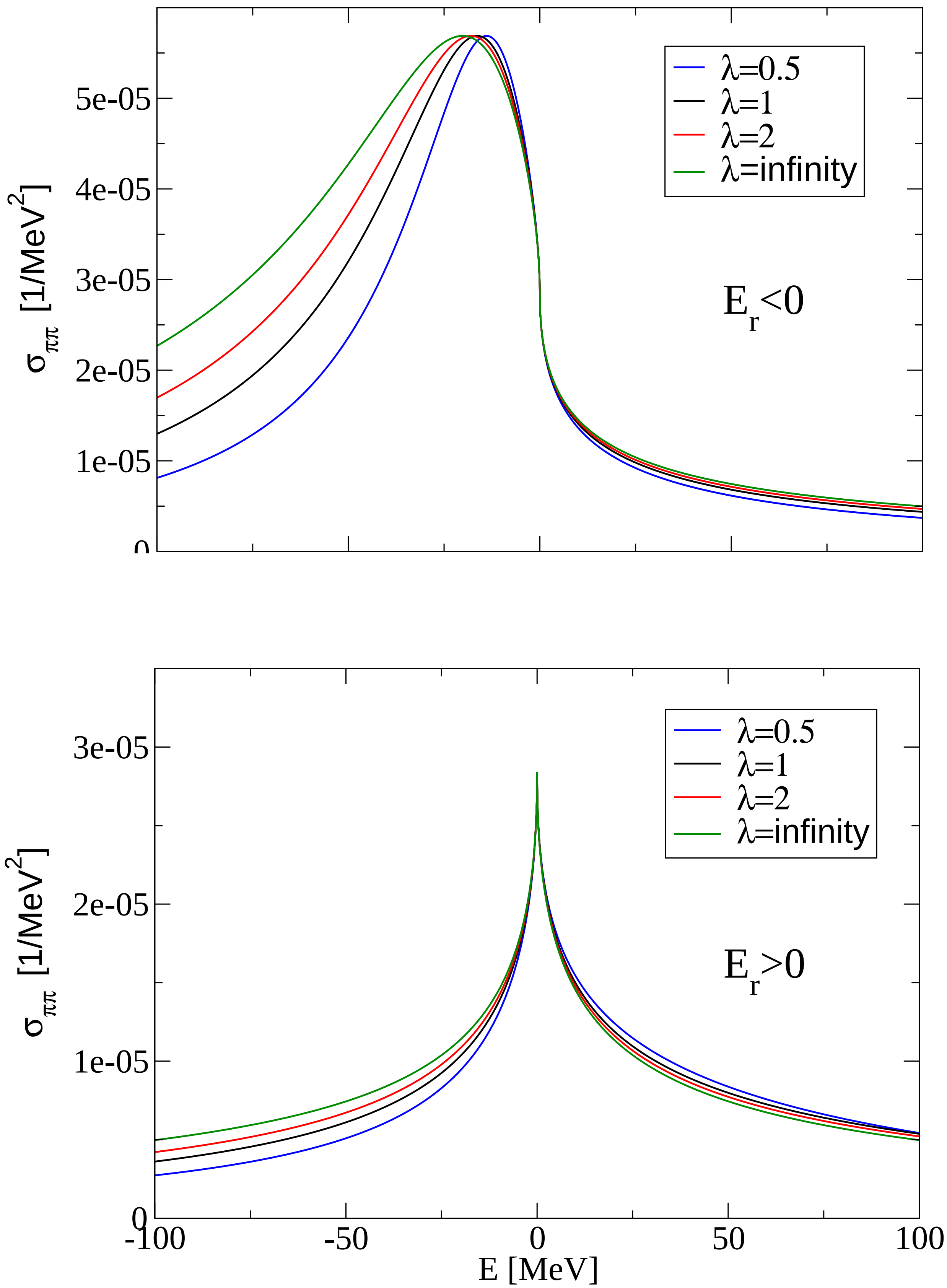
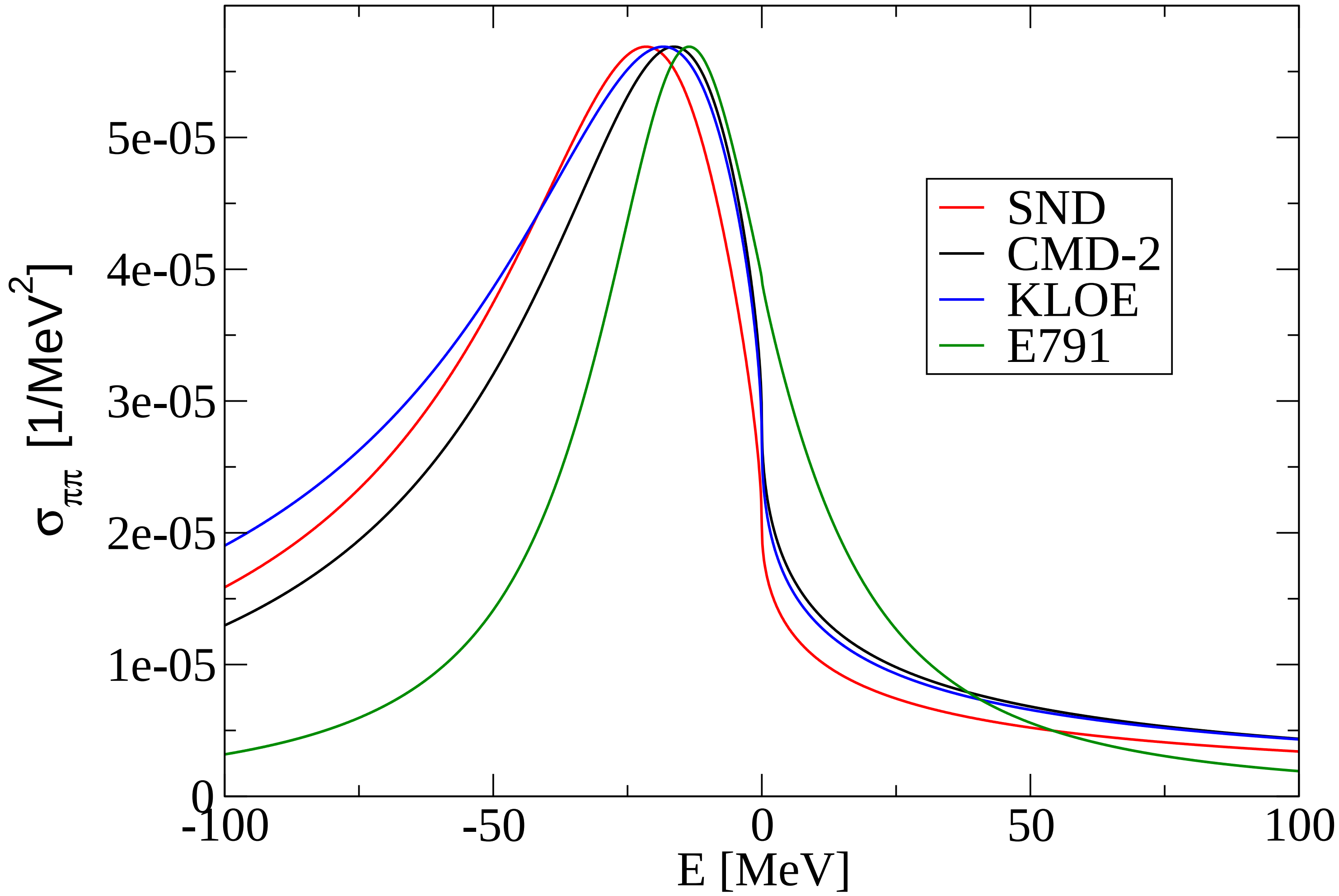
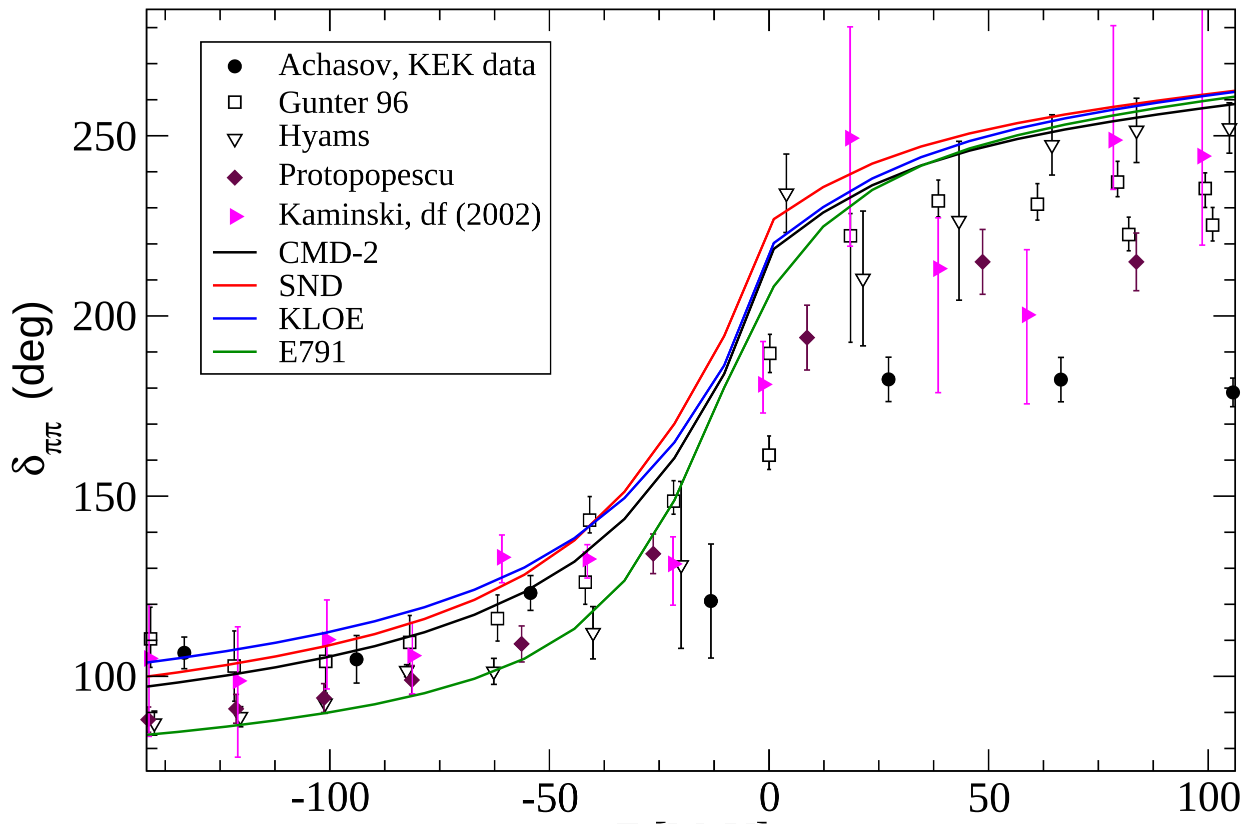
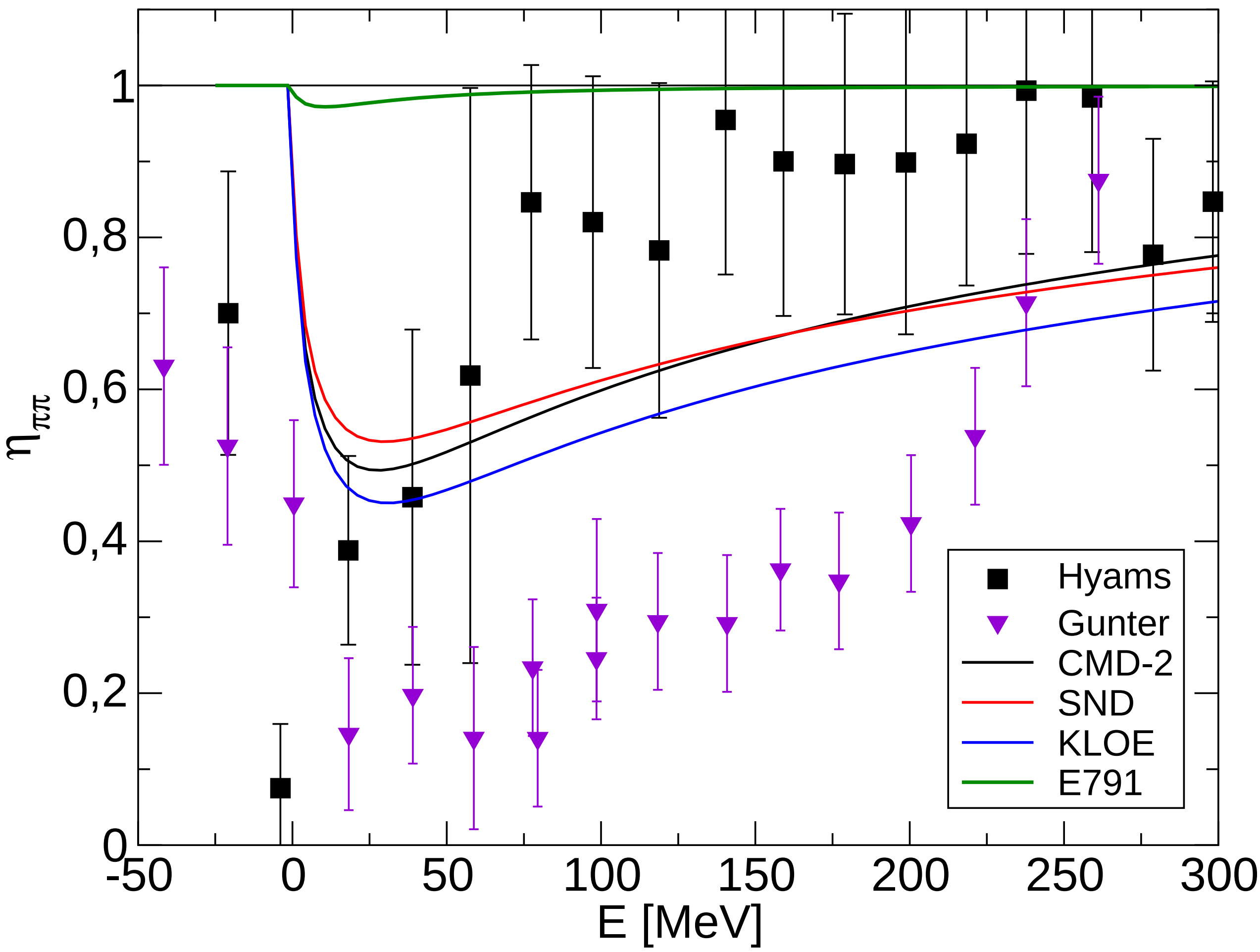


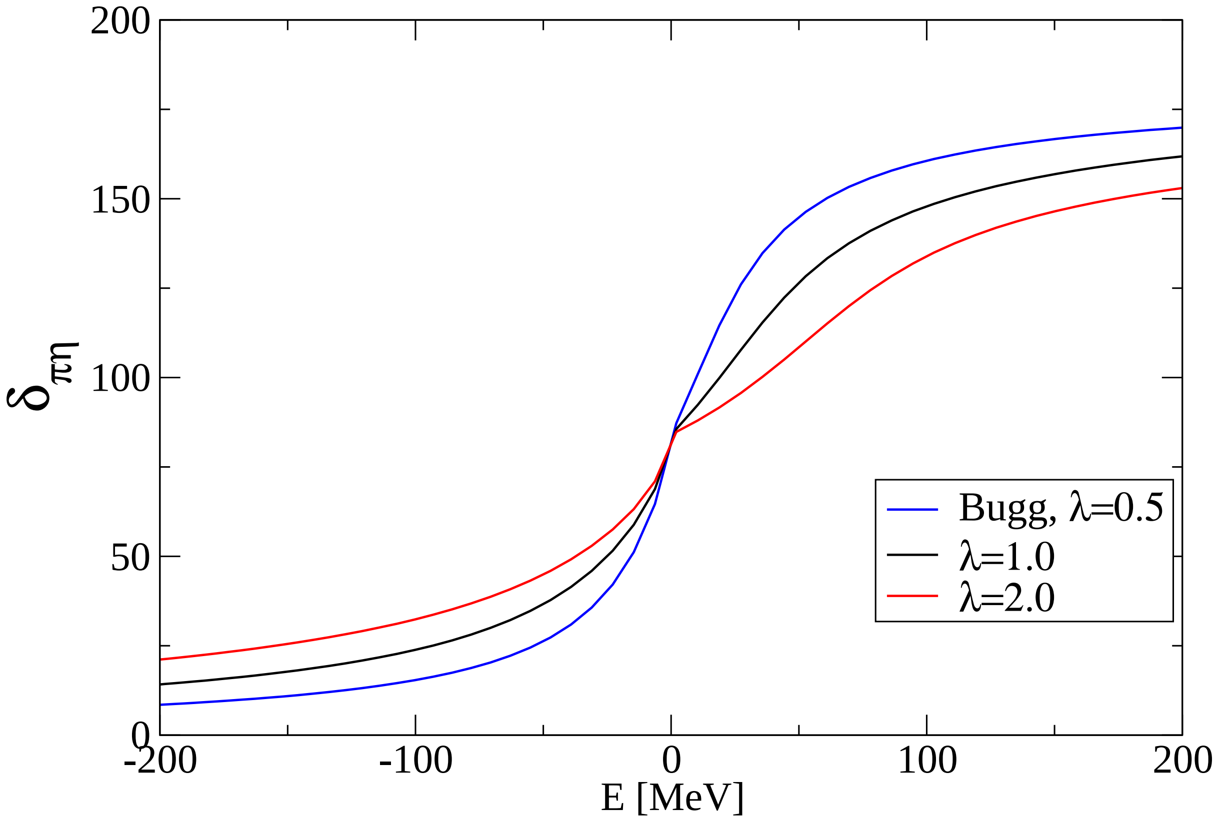
Figure 2: Results for the $\pi\pi$ cross section. The curves are results based on Flatté distributions taken from the table.

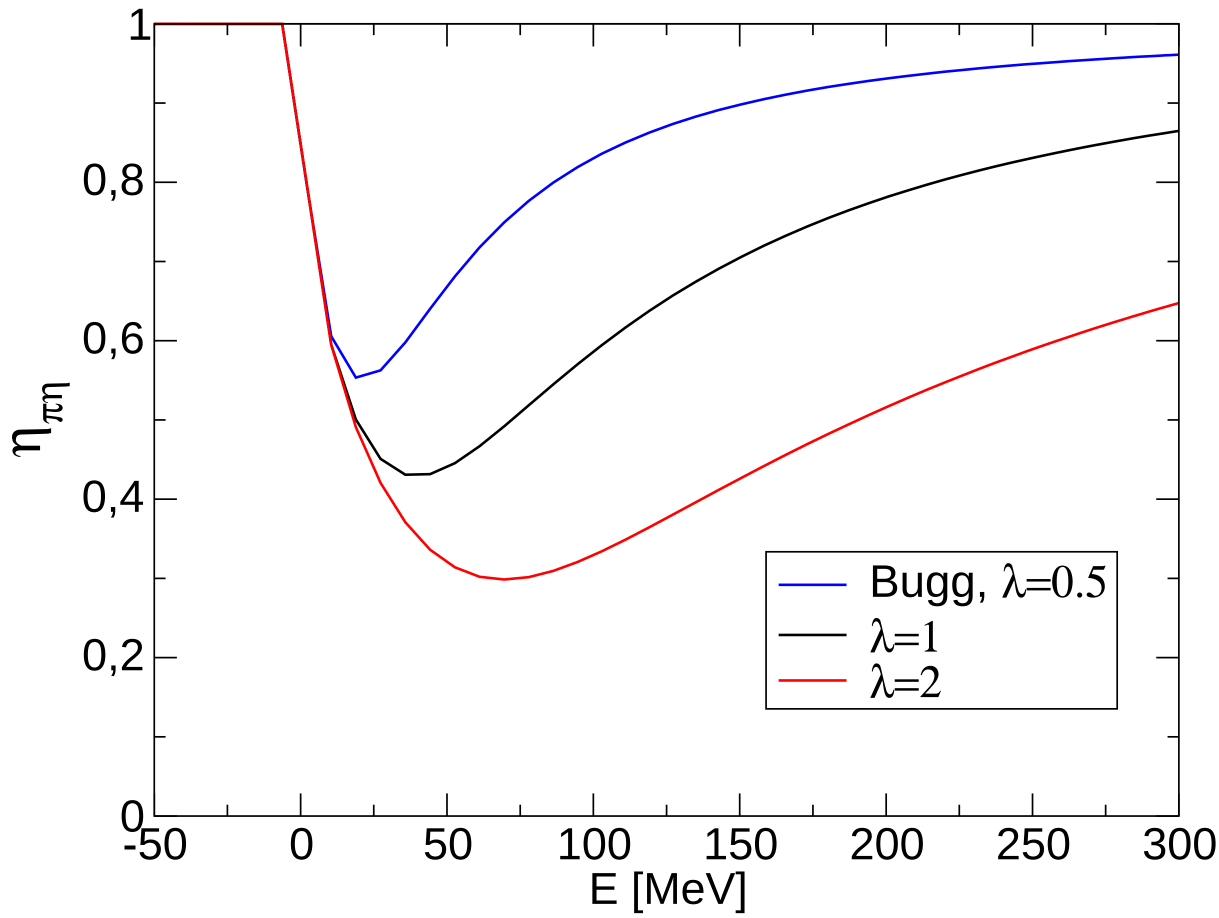



Achasov, KEK data
Gunter 96
Hyams
Protopopescu
Kaminski, df (2002)
CMD-2
SND
KLOE
E791
$\delta_{\pi\pi}$ (deg)
250
200
150
100
-100
-50
0
50
100

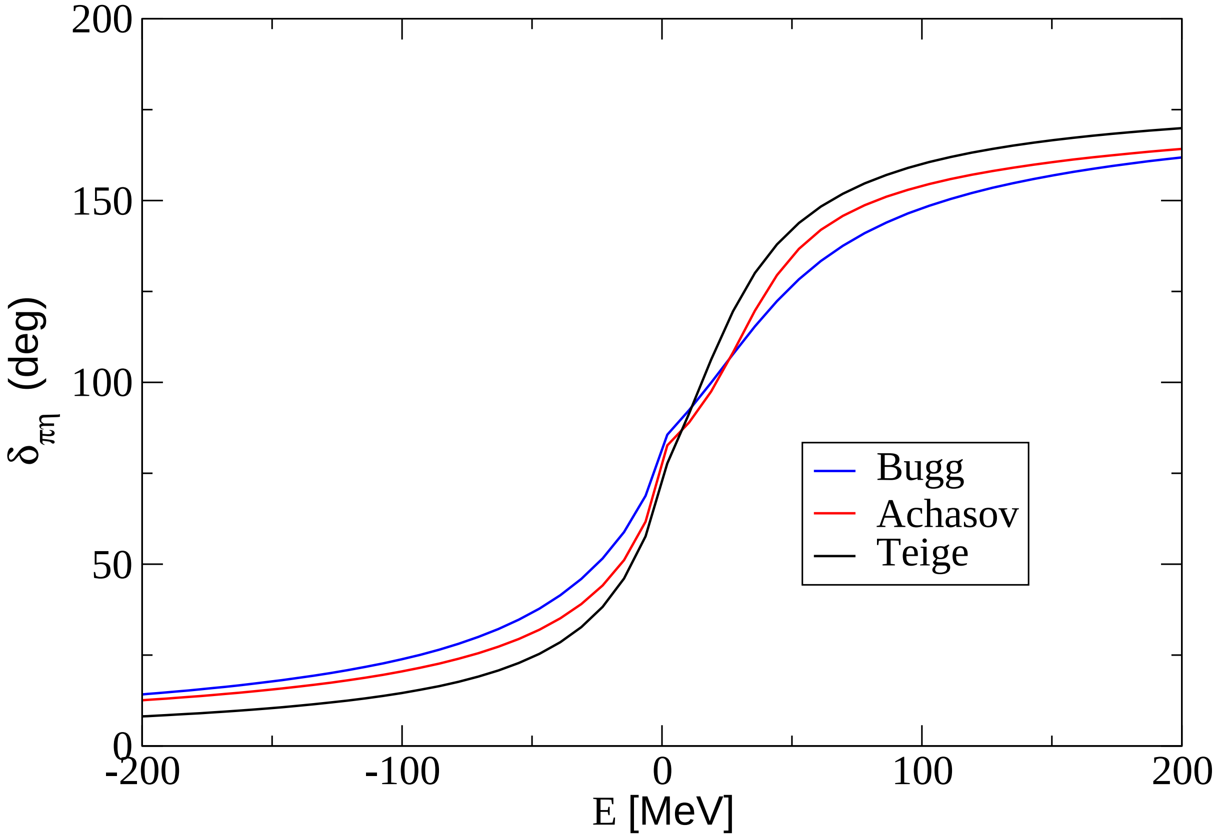

Hyams
Gunter
CMD-2
SND
KLOE
E791
$\eta_{\pi\pi}$
1
0,8
0,6
0,4
0,2
0
-50
0
50
100
150
200
250
300
E [MeV]

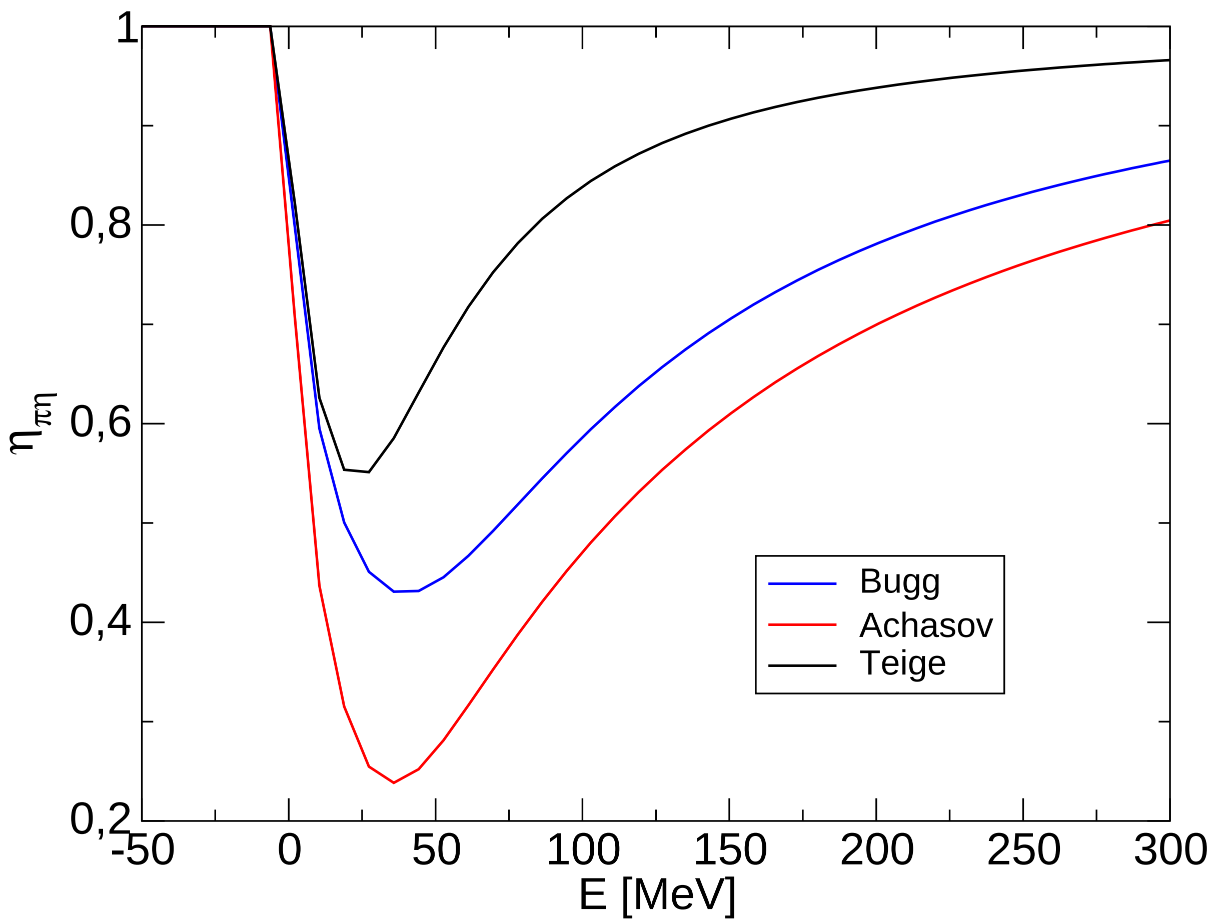
Scaling tendency





πη results based on Flatte parameters from the table





Conclusions

We studied properties of the Flatté and Flatté-like distributions, which are usually employed to describe S-wave resonance-like structures, located near a threshold.

- The experimental observables near threshold are not sensitive to all Flatté parameters (E_{BW}, $\bar{g}_P$, $\bar{g}_K$) but only to the two ratios $R = \bar{g}_K/\bar{g}_P$ and $\alpha = 2E_{BW}/\Gamma_P$. There is a large uncertainty in the absolute values of the coupling constants in the literature, whereas the ratios R and α can be extracted from experiments with much better accuracy.

- The scattering length $a_{K\bar{K}}$ is expressed in terms of R and α and therefore can be determined!

- The energy region where the scaling behavior is pronounced is much less for the a_0 meson as compared to the f_0.

- In principle, only the information about all Flatté parameters opens the possibility to calculate the $K\bar{K}$ effective range parameters and to reconstruct the position of the poles of the scattering amplitude in the complex k plane. The knowledge of the position of the poles allows to draw conclusions on the nature of the resonance.

(V.Baru et al., Phys. Lett. **B 586**, 53 (2004))

However, the ratio R is also an interesting quantity. For example, a large R is a strong indication for a molecular-like structure of the near-threshold resonance.