TBILISI AUTUMN LECTURES 2013

D. Gotta

exercises solutions

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1 Radiation and particles

1-1 Prove that $E = mc^2$ is equivalent to $E = \sqrt{p^2c^2 + m_0^2c^4}$.
Inserting $p = m\beta c$ *with* $m = \gamma m_0$ *and* $\gamma = 1/\sqrt{1-\beta^2}$ *yields* $E = \gamma m_0 c^2$.

1-2 A typical X-ray energy of medical devices is 20 keV.
a) Calculate the wavelength of the radiation.
From $E = h\nu$ *and* $c = \lambda \cdot \nu$ *we obtain* $\lambda = \frac{2\pi\hbar c}{E} \approx \frac{2\pi 200\, MeV fm}{20\, keV} = 20\pi\, pm$.
b) Which acceleration voltage U is needed to produce electrons of the same wavelength?
The de Broglie relation $\lambda = \frac{h}{p}$ *yields* $p = \frac{2\pi\hbar c}{\lambda c} = 200\, keV/c$ *(use* $\hbar c = 200\, MeV \cdot fm$*). The kinetic energy is non-relativistic (relativistic)* $T = \frac{p^2}{2m_e} = 39.1\, keV$ *(*$T = E - m_e = \sqrt{p^2c^2 - m_e^2} - m_e = 37.8\, keV$*). Hence,* $U = 39.1\, kV$ *(38.8 kV).*

1-3 a) Calculate the wavelength of an electron in the H-atom ground state.
Non-relativistic $T_{kin} = \frac{m}{2}v^2 = \frac{mc^2\alpha^2}{2}$*, i.e.,* $\beta = \alpha$*. Hence,* $\lambda = \frac{2\pi\hbar}{p} = \frac{2\pi \cdot 200\, MeV \cdot fm}{(511/137)\, keV} = \frac{4\pi \cdot 10^{-10}\, m}{3{,}73} \approx \pi \cdot 10^{-10}\, m$.
b) Assume the wave length to be equal the orbit length of electron (wrong classical picture). What is the radius of the H atom?
$\lambda = 2\pi \cdot a_B$ *yields* $a_B = 0.5 \cdot 10^{-10}\, m$.
c) Estimate the dimension of an H atom by using the uncertainty relation $\Delta x \cdot \Delta p = \hbar$.
The orbit velocity $v = \alpha \cdot c$ *yields* $\Delta x = \frac{\hbar c}{m_e \cdot \alpha \cdot c} = \frac{200\, MeV fm}{(511/137)\, keV} = \frac{2 \cdot 10^{-10}\, m}{3{,}73} \approx 0.5 \cdot 10^{-10}\, m$*.* Δx *is rather the diameter than the radius. By using the velocity projection, i.e., the average over one half period* $[0,\pi]$ $\int sinx\, dx = 2/\pi$*, one obtains* $a_B \approx \frac{2}{\pi}\Delta x$*.*

1-4 Estimate by using the uncertainty relation:
a) The mass of the virtual particle mediating the nuclear force, where the average nucleon separation is 1.3 fm.
Virtual particles are assumed to have $v = c$ *but a momentum of* $\Delta p = m_0 c$ *(*$\hbar c = 197.3\, MeV \cdot fm \approx 200\, MeV \cdot fm$*).*

$$m = \frac{\hbar}{1,3\, fm \cdot c} \approx \frac{200\, MeV \cdot fm}{1,3\, fm \cdot c^2} \approx 150\, MeV/c^2.$$

b) The life time of a strongly decaying particle, *e.g.*, the ρ meson is of the order of $\Delta E = 150\,\text{MeV}$.
For the conjugated variables E and t hold the uncertainty relation $\Delta E \Delta t \approx \hbar/2$ ($c = 30\,cm/ns$).

$$\Delta t = \frac{\hbar c}{\Delta E c}/2 \approx \frac{200\,MeV \cdot fm}{2 \cdot 150\,MeV \cdot 0,3\,m/ns} \approx 0,2 \cdot 10^{-23}\,s.$$

1-5 The radius of a hydrogen atom is of the order of 0.5 Å, the one of the proton of 0.8 fm.
Estimate the order of magnitude of the collision cross section for atomic - atomic and nucleon – nucleon collisions, respectively (1 Å= 10^{-10} m).

$$\sigma_{tot,atom} = 4 \cdot \pi r_{atom}^2 = 4 \cdot \pi \cdot 0.25 \cdot 10^{-20} m^2 = \pi \cdot 10^{-16}\,cm^2 = \pi \cdot 10^8\ b$$
$$\sigma_{tot,nucl} = 4 \cdot \pi r_{nucleon}^2 = 4 \cdot \pi \cdot 0.64 \cdot 10^{-30} m^2 \approx 8 \cdot 10^{-26}\,cm^2 = 80\,mb$$

The unit of the cross section in the subatomic regime is usually given in barn(b) with $1\,b = 10^{-24}\,cm^2$. Estimates coincide at low energies with literature values to about a factor of two.

1-6 The electromagnetic force is mediated by virtual phontons of mass zero. Derive by using the uncertainty relation the distance law of the electric force.
For $m = 0$ particles $p = \frac{E}{c}$ holds. Using the uncertainty relation $E \cdot t = \hbar$ and $c = \frac{x}{t}$ one obtains $p = \frac{\hbar}{c} \cdot \frac{1}{t}$. The force of coupling strength g is given by

$$F = g \cdot \frac{dp}{dt} = -g \cdot \frac{\hbar}{c} \cdot \frac{1}{t^2} = -g \cdot \hbar c \cdot \frac{1}{x^2}$$

Comparing with the Coulomb law, we find the coupling strength $\alpha = \frac{1}{137}$ of the electromagnetic force because of $\alpha \cdot \hbar c = \frac{e^2}{4\pi\epsilon_0}$ with the distance law of the force and the potential $V = \int F \cdot dr$

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{r^2}$$
$$V = -\frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{r}$$

2 Interaction of radiation with matter

2-1 The maximum absorbance of the medium wavelength cone cells in the human eye is reached at 534 nm (green), where the maximum of the sun's emission spectrum at sea level is 550 nm. What is the energy of the corresponding photon?

$$E = h\nu = \frac{2\pi\hbar c}{\lambda c} = \frac{2\pi \cdot 200\, MeV \cdot fm}{550\, nm \cdot 0.3\, cm/ns} = 2.25\, eV$$

2-2 The linear attenuation coefficient μ of 20 keV and 50 keV X-rays is in water 0.6/cm and 0.2/cm, respectively.
a) Which fraction of the radiation is absorbed, before a 10 cm deep lying structure is reached.
Transmission for 20 keV (50 keV) radiation is given by $T = e^{-\mu\cdot d} = e^{-6} = 0.0025$ *(*$e^{-2} = 0.135$*). The aborbed fraction is* $A = 1 - T = 99.75\%$ *(86.5%).*
b) Which thickness of a lead sheet is needed to absorb the same fraction of 20 keV radiation as a 10 cm thick water layer ($\mu_{lead} = 1015/cm$)?
One can also equate the transmission, i.e.,

$$e^{-0.6/cm\cdot 10\, cm} = e^{-1015/cm\cdot x_{lead}}$$
$$x_{lead} = \frac{6}{1015}\, cm = 60\, \mu m.$$

2-3 A low concentration suspension of milk in water is irradited by white light. What colour impression is observed at the wall of the glas container after transmission and at the side walls?
The colour impression is reddish after transmission and blueish at the side walls.

2-4 Order the processes contributing to the interaction of electromagnetic radiation by their magnitude for energy 1 keV and 1 GeV.
1 keV: photo effect > Rayleigh scattering > Compton scattering
1 GeV: pair production at the nucleus > pair production at the electrons > Compton scattering

2-5 Is it possible to measure X-rays of 6 keV by using gaseous detectors?
The charge created by the photo electron is amplified in Geiger-Müller counters or wire chambers.

3 Interaction of particles with matter

3-1 In the "Bethe-Bloch" range the minimum stopping power S_{min} in hydrogen (H_2) of 4.10 MeVg^{-1}cm^2 is reached at $\beta\gamma = 4$.
a) Calculate the corresponding kinetic energy in units of the projectile rest energy m_0c^2.

$$(\beta\gamma)^2 = \frac{\beta^2}{1-\beta^2} = 16 \quad \Rightarrow \quad \gamma = \sqrt{17}$$
$$T = E - m_0c^2 = (\gamma - 1)m_0c^2 \quad \Rightarrow \quad T/m_0c^2 = \sqrt{17} - 1 = 3.12$$

b) What is the value of S_{min} for D_2?

$$S \propto \frac{Z_{target}}{A} \quad \Rightarrow \quad S_{D_2} = \frac{1}{2} S_{H_2} = 2.05\ MeV g^{-1} cm^2$$

3-2 Estimate (non relativistic) relative to the protons energy loss (dE/dx) and kinetic energy
a) the energy loss of deuterons (d) having the same kinetic energy than protons (p) (assume $m_d = 2 \cdot m_p$).

$$\left(\frac{dE}{dx}\right) \propto \frac{1}{v^2},\ i.e.,\ \left(\frac{dE}{dx}\right) \propto \frac{m}{T_{kin}}. \tag{1}$$

Hence, for $T_p = T_d$ one obtains

$$\left(\frac{dE}{dx}\right)_p = \frac{m_p}{m_d}\left(\frac{dE}{dx}\right)_d \approx \frac{1}{2}\left(\frac{dE}{dx}\right)_d. \tag{2}$$

b) the kinetic energy of deuterons experiencing the same energy loss than protons.

For $v_p = v_d$, or equivalent the same time–of–flight, one obtains

$$T_d = \frac{m_d}{m_p} \cdot T_p \approx 2\,T_p. \tag{3}$$

3-3 An aluminum window causes angular straggling of the penetrating particles. By which factor can the angular resolution be improved by changing to a beryllium window of the same effective thickness?

Because of $\bar{\Theta} \propto \sqrt{\frac{Z^2}{A}}$, the factor is $\sqrt{\frac{13^2}{27} \cdot \frac{9}{4^2}} \approx 1.9$.

3-4 A $\Delta E - E$ arrangement has the following properties. The thickness of the ΔE (first) counter corresponds to a proton range of 10 MeV, the total thickness of ΔE and E counter to 100 MeV protons.
a) In your experiment, you expect a continuum of proton energies up to 200 MeV together with a strong contribution of monoenergetic protons at 200 MeV. Sketch the plot ΔE versus $\Delta E + E$ plot including numbers of outstanding points.

The hyperbola starts at $\Delta E, \Delta E + E = (10, 10)$ *MeV from the end of the straight line containing all protons with E<10 MeV stopped in the first counter. From the Bethe-Bloch formula we read the approximations*

$$\left(\frac{dE}{dx}\right) \propto \frac{z^2_{projectile}}{T_{kin}}$$

$$R = \int \frac{dT_{kin}}{\left(\frac{dE}{dx}\right)} \propto \int \frac{T_{kin} dT_{kin}}{m} \propto \frac{T^2_{kin}}{m}$$

Hence, ΔE*(100 MeV)≈ 1 MeV and* ΔE*(200 MeV)≈ 0.5 MeV. The 200 MeV protons have still more than* $\frac{3}{4}$ *of the initial energy when leaving the* $\Delta E - E$ *stack, i.e., about 40 MeV energy are deposited in the stack.*

As an example, we give exact values for PILOT B plastic scintillator (ρ*=1.02 g/cm*^{3}*):*
R(10 MeV)=1.2 mm, R(100 MeV)=75.2 mm, R(200 MeV)=254 mm
ΔE *in first counter:* ΔE*(100 MeV) =0.90 MeV,* ΔE*(200 MeV) =0.55 MeV.*
The energy of the 200 MeV protons when leaving the stack is found by looking up the energy for the difference of the ranges for 100 and 200 MeV being 179 mm, which is found to be 163 MeV. Therefore, the 200 MeV peak is located at $\Delta E, \Delta E + E = (0.55, 37)$ *MeV.*

b) By misadjustment of the beam, deuterons and alpha particles are produced from the support materials of the target. Where appear these particles in the $\Delta E - E$ plot?

Because of $R \propto \frac{T^2_{kin}}{m}$ *Deuterons (*$m_d = 2 \cdot m_p$*) up to about* $100 \cdot \sqrt{2} \approx 140$ *MeV are stopped in the stack. As* $v_d = v_p/\sqrt{2}$*, the deuteron hyperbola lies above the one of the protons.* α *particles are much more separated and above the protons and deuterons, because of the* $z^2_{projectile}$*-dependence of* $\left(\frac{dE}{dx}\right)$*.*

4 Detection concepts

4-1 What is your count rate estimate for proton - carbon collisions when using a 1 μm thick carbon foil and a beam intensity of 10^6 protons per second (assume a radius - mass dependence of $r = 1.3\,fm \cdot A^{1/3}$ and $\rho_{carbon} = 2\,g/cm^3$)?

The cross section can be estimated as follows:
With $r_C \approx 1.3\,fm \cdot 12^{1/3} \approx 3\,fm$ *(literature value for carbon charge radius is 2.5 fm) one obtains* $\sigma_{pC} \approx \pi \cdot (3+0.8)^2\,fm^2 \approx 450\,mb$ *(literature value* $\sigma_{pC} \approx 260\,mb$*).*

$$\begin{aligned} \Delta N &= N_0 \cdot \rho \cdot \frac{N_A}{A} \cdot \sigma \cdot \Delta x \\ &= 10^6/s \cdot 2\,g/cm^3 \cdot \frac{6 \cdot 10^{23}}{12\,g} \cdot 450\,mb \cdot 10^{-4}\,cm \\ &= 4.5/s \end{aligned}$$

4-2 Neutral pions of mass 135 MeV/c^2 decay via $\pi^0 \rightarrow \gamma\gamma$.
a) Which kind of detector material and geometry is suitable to measure low-energy π^0?

The two γ *particles of energy close to* $m_{\pi^0 c^2}/2$ *are emitted (almost) back to back. Anorganic scintillaors like NaI(Tl) or CsI(Tl) with sufficient light conversion efficiency are suitable. As the momentum is usually isotropic in space, an approximate* 4π *geometry is preferable.*
b) How very high energetic π^0 must be detected?

At high energies, shower capable devices are needed. Because of momentum conservation, the two photons are emitted close to forward.

4-3 A multi wire proportional chamber (MWPC) of 1 cm thickness is filled with argon gas of 1 bar ($\rho_{NTP} = 1.6 \cdot 10^{-3}\,\mathrm{g/cm^3}$).
a) Estimate the theoretically achievable energy resolution of the chamber for minimum ionizing particles assuming an almost perpendicular crossing (assume $W_{i,Ar} = 26\,eV$ to produce one electron-ion pair).

The specific energy loss of minimum ionizing particles of about 2 MeV/gcm^{-2} *leads for a typical gas density of* 10^{-3} *g/cm*^{3} *to an energy deposit of*

$$E = \left(\frac{dE}{dx}\right)_{MIP} \approx 2\,\mathrm{MeV/gcm^{-3}} \cdot 1.6 \cdot 10^{-3}\mathrm{g/cm^3} = 3200\,\mathrm{eV/cm} \qquad (4)$$

leading to a primary ionisation of $n_p = E/W_i = 3200\,eV/26\,eV \approx 120$, *where the average energy to produce an electron – ion pair is* $W_{i,Ar} = 26\,eV$. *This suggests an energy resolution in terms of one standard deviation of* $\sigma = \sqrt{125}$ *or a relative resolution of* $\sigma/n_p = 9\%$, *i. e., 280 eV (1*σ*). Including, however, the Fano factor for argon,* $F_{Ar} = 0.2$, *we obtain a modified primary ionization of* $n_p^{,} = F \cdot n_{primary} \approx 25$, *i. e.,*

$$\sigma = \sqrt{\frac{F_{Ar} \cdot E}{W_{i,Ar}}} = \sqrt{25} \quad \text{or} \quad \sigma/n_p^{,} = 1\sqrt{25} = 20\%, \qquad (5)$$

corresponding to 640 eV (1σ*) for 3.2 keV energy deposit.*

For practical reasons, detector resolutions are usually given by the full width half maximum (FWHM) of the resolution function, which reads $FWHM = 2.35 \cdot \sigma$ *assuming Gaussian distributions.*

b) Compare with the energy resolution of to a xenon high-pressure chamber operated at 4 bar ($\rho_{NTP} = 5.6 \cdot 10^{-3}\,\mathrm{g/cm^3}$ and $W_{i,Xe} = 22\,eV$).

For xenon at 4 bar, $W_{i,Xe} = 22\,eV$ *leads to* $n_{primary}^{,} = F_{Xe} \cdot \frac{44000}{22} = 500$ *when using* $F_{Xe} = 0.25$. *Therefore, we obtain*

$$\sigma/n_p^{,} = \sqrt{\frac{W_{i,Xe}}{F_{Xe} \cdot E}} = \frac{1}{\sqrt{500}} = 4.5\%, \qquad (6)$$

which constitutes an improvement by a factor of almost 5 for the relative energy resolution.

4-4 The fixed dead time of a detector element is 10 μs. What is the maximum particle rate the element can handle, if at least 99% of the particles should be registered?

One distinguishes between paralyzing (paralyzable) and non-paralyzing (non-paralyzable) recording systems. In the first case, an event occuring during the dead time initiated by the preceeding one prolongs the dead time until the end of the second process. In the latter case, the second event is ignored, but would be recorded immediately after the insensitive period caused by the the first event. With R, R', and τ being the true event rate, the measured event rate, and τ the insensitive period of the recording system (dead time) the relation for the paralyzing and non-paralyzing type reads

$$R' = R \cdot e^{-R\cdot\tau} \qquad \text{and} \tag{7}$$

$$R = \frac{R'}{1 - R' \cdot \tau}, \tag{8}$$

respectively. Given R'/R = 99%, one obtains R = 1005/s (1010/s) and R' = 995/s (1000/s) for the paralyzing (non-paralyzing) case.

4-5 The impact point of neutrons from the reaction $\pi^{-3}He \rightarrow dn$ with stopped pions is measured in a plastic scintillator rod of 2 m length and cross section of $5 \times 5\,\mathrm{cm}^2$ from the difference of the light arrival time by using photo multiplier tubes (PMT) at each end. (*Use* $m_\pi = 140\,MeV/c^2$, $m_n = 940\,MeV/c^2$, $m_d = 1876\,MeV/c^2$, $m_{^3He} = 2808\,MeV/c^2$, and $c = 0.3\,\mathrm{m/ns}$.)

a) Sketch the pulse height spectra measured by means of the PMTs.

b) Calculate maximum and minimum neutron time-of-flight (TOF) to the rod placed in 4 m distance perpendicular to the direction target - center of the rod.

Energy and momentum conservation yield

$$\begin{aligned} E_d + E_n &= Q + (m_d + m_n) = m_X \\ p_d &= p_n \qquad \text{and} \\ Q &= T_d + T_n = (m_\pi + m_{^3He}) - (m_d + m_n) = 132\,\mathrm{MeV}, \end{aligned}$$

where the Q value of the reaction is the sum of final state kinetic energies T. We introduce $m_X = m_\pi + m_{^3He} = Q + (m_d + m_n)$.

Next we estimate $\beta = \frac{p}{E}$ to decide for or against a relativistic calculation. As $m_d \approx 2m_n$), $T_n \approx 2T_d \approx 88\,MeV$, i.e., $\beta \approx \frac{\sqrt{2m_nT_n}}{T_n+m_n} \approx \frac{\sqrt{2\cdot1000\cdot100}}{1000} = \frac{\sqrt{20}}{10} \approx$

0.45. *Hence, we should use relativistic kinematics.*

$$
\begin{aligned}
E_d^2 &= [E_n - m_X]^2 \\
&= p_d^2 + m_n^2 = E_n^2 + m_d^2 - m_n^2 \\
E_n &= \frac{m_X^2 - m_d^2 + m_n^2}{2m_X} \quad = \frac{(Q + m_d + m_n)^2 - m_d^2 + m_n^2}{2(Q + m_d + m_n)} \\
T_n &= \frac{(m_X - m_n)^2 - m_d^2}{2m_X} = \frac{Q}{2} \cdot \frac{Q + 2m_d}{Q + m_d + m_n}
\end{aligned}
$$

We obtain $E_n = 1027\,MeV$ *and* $p_n = 413.6\,MeV/c$ *yielding* $\beta = 0.403$. *The minimum TOF is* $\frac{4\,m}{0.3\,m/ns} = 13.3\,ns$. *Maximum TOF is* $\frac{\sqrt{17}\,m}{0.3\,m/ns} = 13.8\,ns$.
c) Which kind of light reaches primarily the PMTs (assume an index of refraction of n=1.5 of the scintillator material)?

Most of the light arriving at the PMTs is totally reflected from the scintillator surface. Given $n = 1.5$ *the critical angle for total reflection is* $\alpha_{total} = arcsin(1/1.5) = 42°$. *For a neutron impact in the center of the rod, the fraction of solid angle covered by one PMT is* $\left(\frac{\Delta\Omega}{4\pi}\right) = \frac{25\,cm^2}{100\,cm^2} = 2.5 \cdot 10^{-5}$, *whereas the cone of all totally reflected light is*

$$
\begin{aligned}
\left(\frac{\Delta\Omega}{4\pi}\right) &\approx \frac{2}{\pi} \cdot \frac{\pi\,[2 \cdot (1 - sin\,\alpha_{total}) + sin^2(\frac{\pi}{2} - \alpha_{total})]}{4\pi} \\
&\approx 17\%.
\end{aligned}
$$

Hence, for a pointlike light source, the light reflected close to α_{total} *is collected most efficiently. The factor* $\frac{2}{\pi}$ *is due to the rod's rectangular cross section.*
d) Assume a time resolution of $\Delta t\Delta = 354\,ps$ *of each PMT. What is the position resolution* Δx *achievable along the rod?*

The light path close to α_{total} *is enlarged by the factor* $\frac{1}{sin(\frac{\pi}{2} - \alpha_{total})}$. *The light velocity in the rod is* $c_{rod} = \frac{c}{1.5}$. *Therefore, the effective light propagation time is*

$$
c_{eff} = \frac{sin(\frac{\pi}{2} - \alpha_{total})}{n} \cdot c = \frac{0.75}{1.5} = 0.50 \cdot c\,,
$$

which corresponds to 13.3 ns for the 2 m long rod. Time resolution is given by folding the two PMTs. Hence, $\Delta x = \frac{\sqrt{2\Delta t^2}}{13.3\,ns} \cdot 2000\,mm = 75\,mm$.