



QUANTUM COMPUTING - BASICS

11.09.2019 | LOTTE GECK

Member of the Helmholtz Association



JÜLICH
Forschungszentrum

CRYOGENIC INTEGRATED CIRCUITS

Recap

Goal

- Design, implement and test scalable control and readout electronics for quantum computers

Challenges

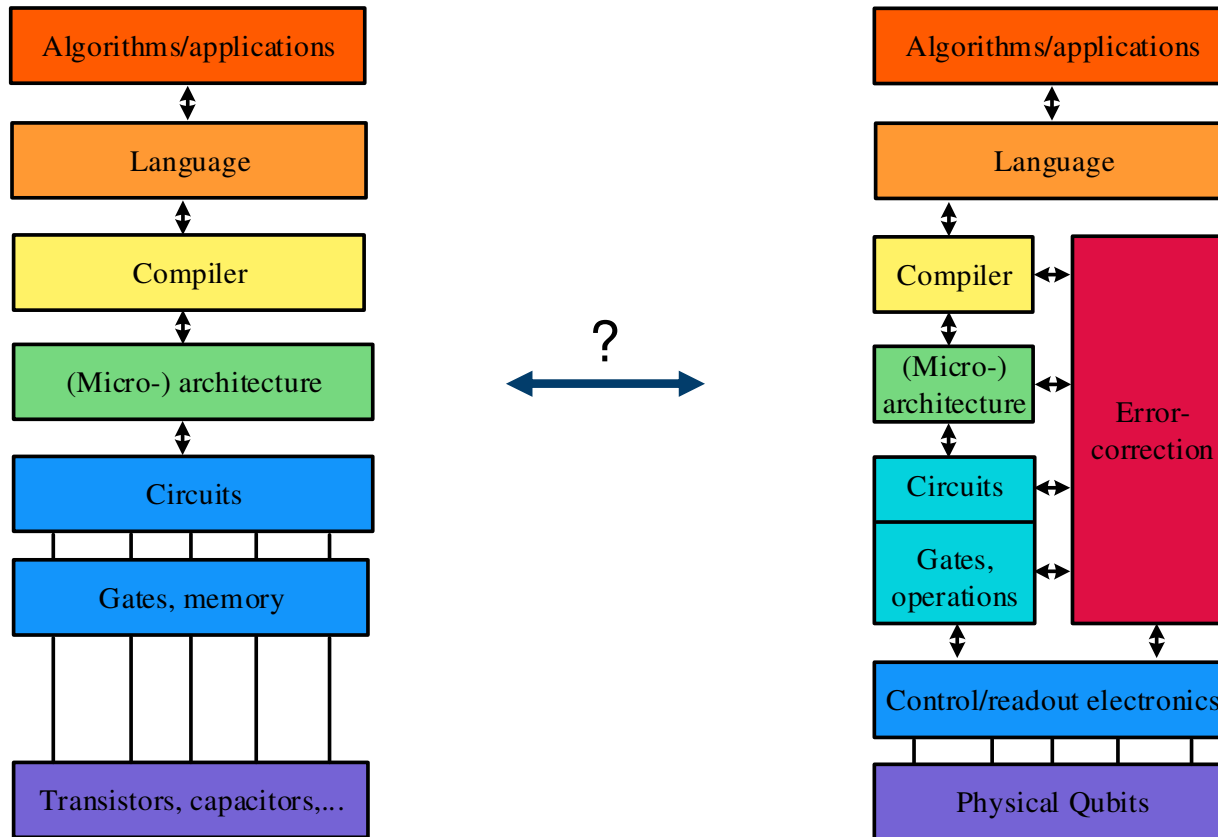
- Scalability: operate thousands and millions of qubits in parallel
- Cryogenic environment with very limited cooling power (few mW in total at $< 100\text{mK}$)
- Area restrictions for 1:1 coupling of electronics with qubits
- Interface to room temperature electronics

➔ **Only possible with highly integrated cryogenic electronics**



oxford-instruments.com

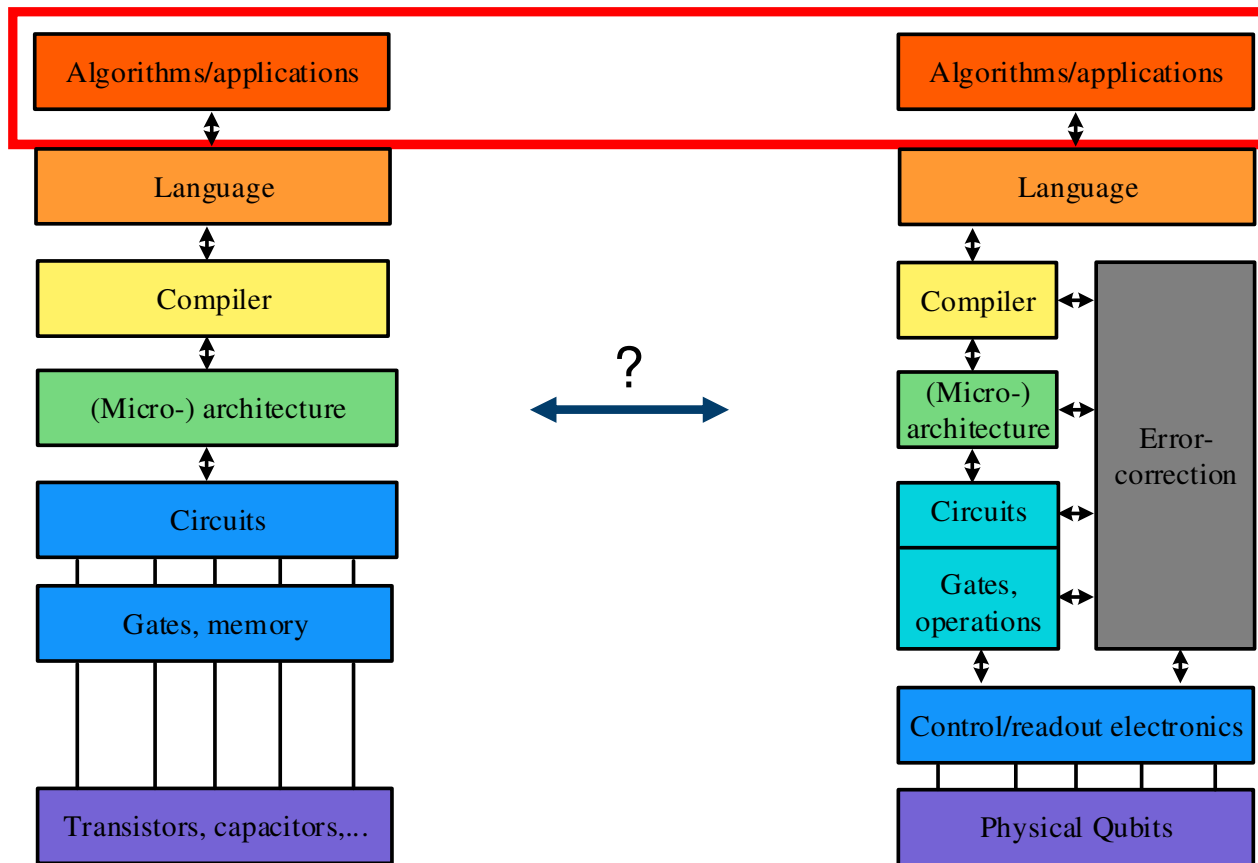
CLASSICAL VS QUANTUM COMPUTER



Mathematicians

Computer Scientists
Electrical Engineers

CLASSICAL VS QUANTUM COMPUTER



QC VS CLASSICAL C

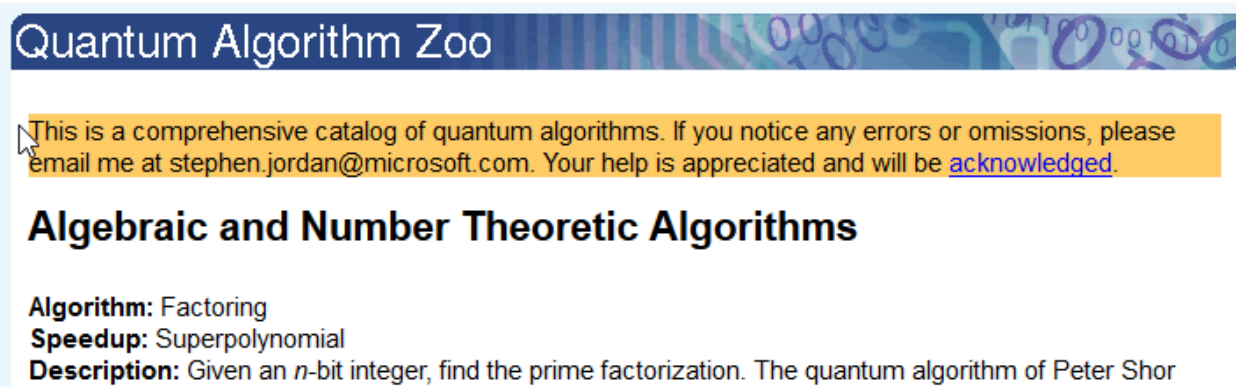
Algorithms

Classical Computer

- Can run classical algorithms

Quantum Computer

- Can run classical and quantum algorithms



Quantum Algorithm Zoo

This is a comprehensive catalog of quantum algorithms. If you notice any errors or omissions, please email me at stephen.jordan@microsoft.com. Your help is appreciated and will be [acknowledged](#).

Algebraic and Number Theoretic Algorithms

Algorithm: Factoring
Speedup: Superpolynomial
Description: Given an n -bit integer, find the prime factorization. The quantum algorithm of Peter Shor

Most efficient algorithm

Run time: sub-exponential

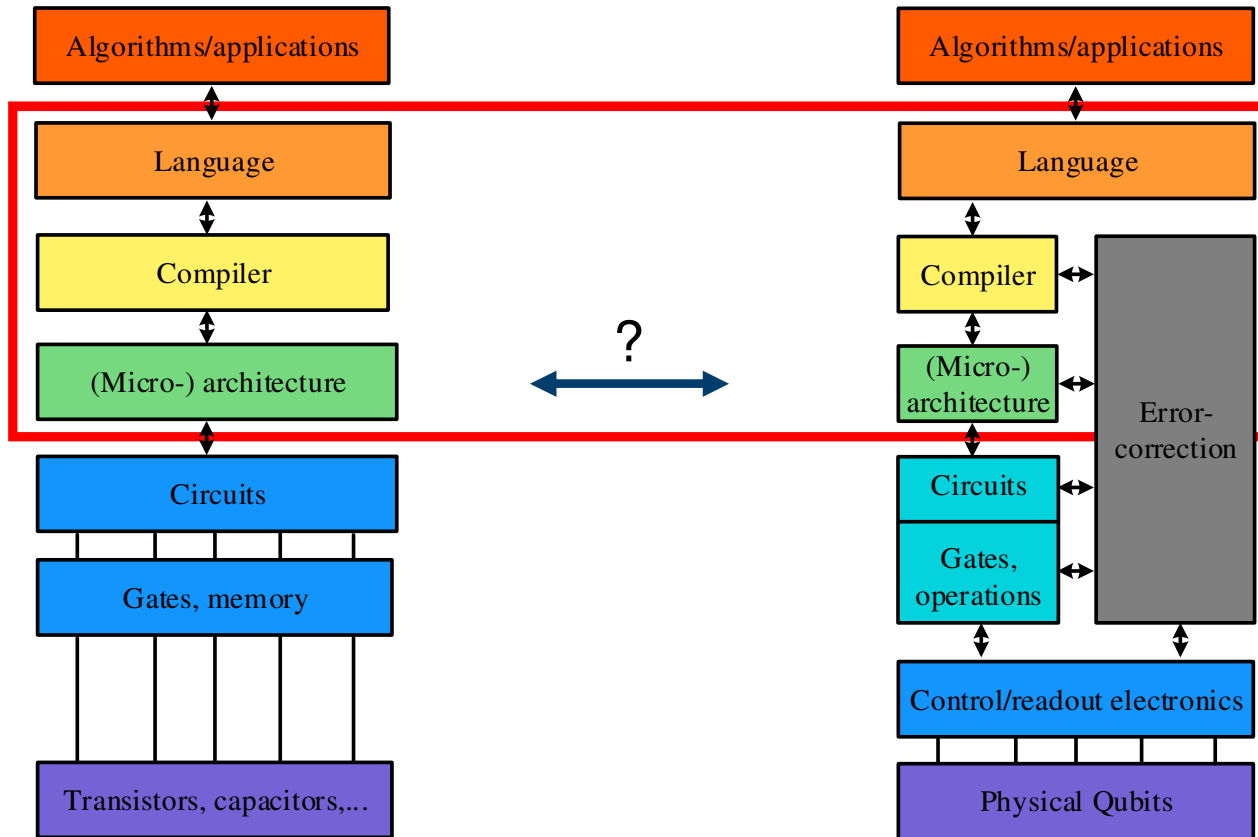
Shor's algorithm

→ Factoring a 2048 bit number takes 100 years on a super computer

→ Factoring a 2048 bit number takes 26.7 hours
~50 algorithms
quantumalgorithmzoo.org

[1]

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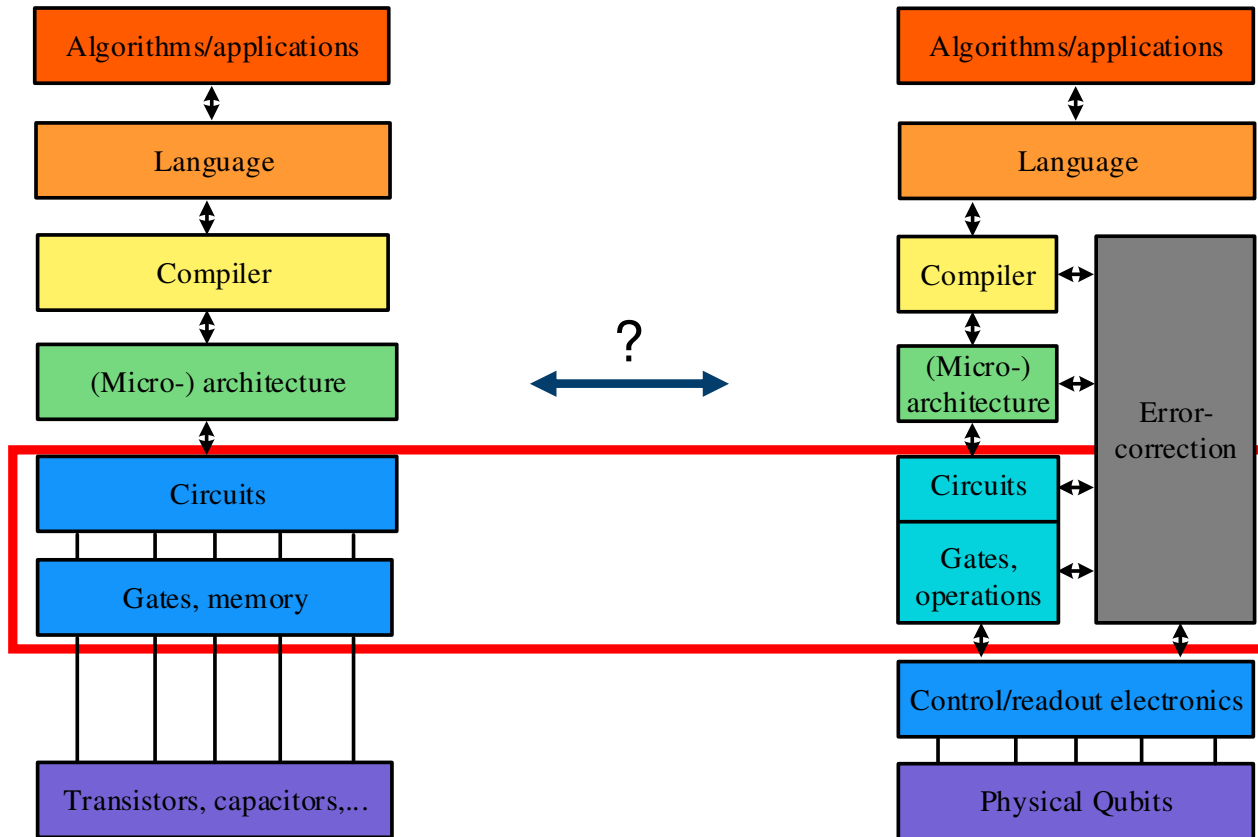


Programming Languages, Compiler, Software, Microarchitecture

- Languages:
 - QCL, Quantum pseudocode, Q#, Qlanguage, OpenQL,...
- Instruction sets:
 - Quil, OpenQASM, QUIZA,...
- Software development kits:
 - ProjectQ, Quiskit, Forest, Quantum Developmet Kit, Cirq,...



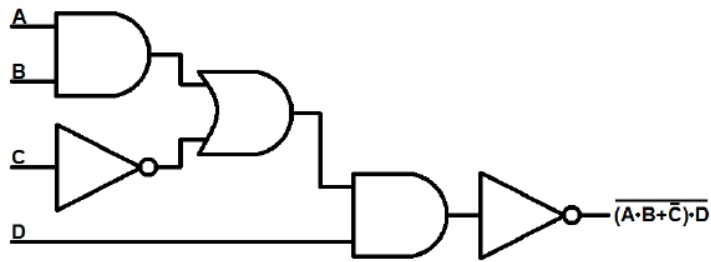
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QC VS CLASSICAL C

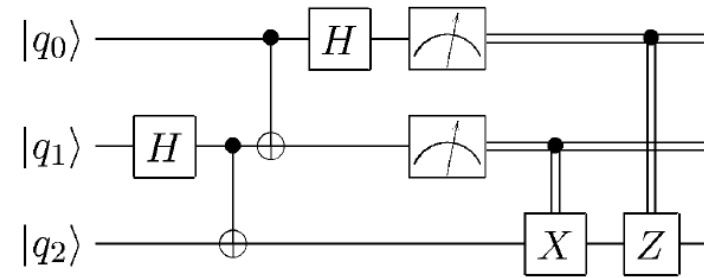
Circuits, Gates, Memory

Classical Computer



Irreversible and deterministic

Quantum Computer



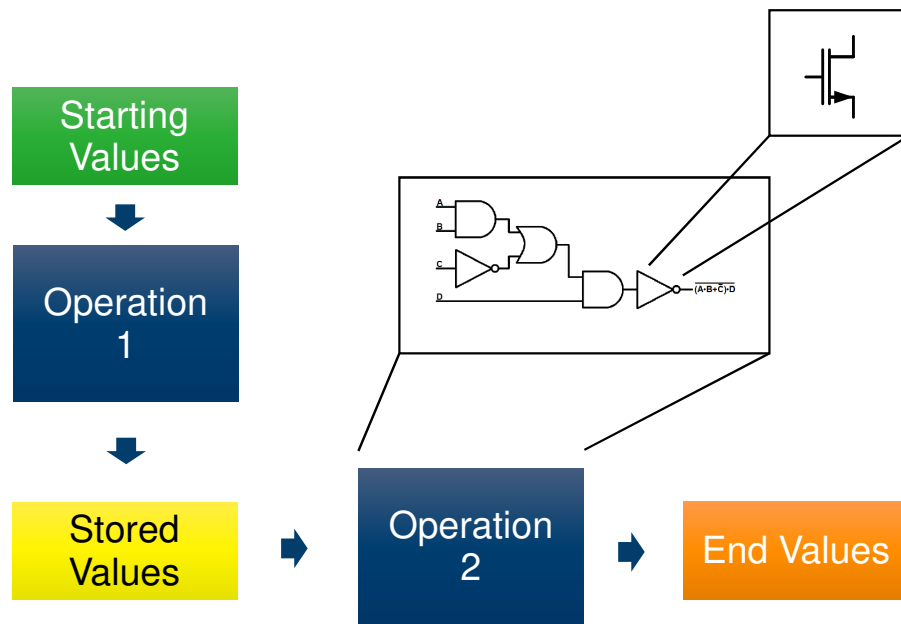
Reversible and non-deterministic

[1]

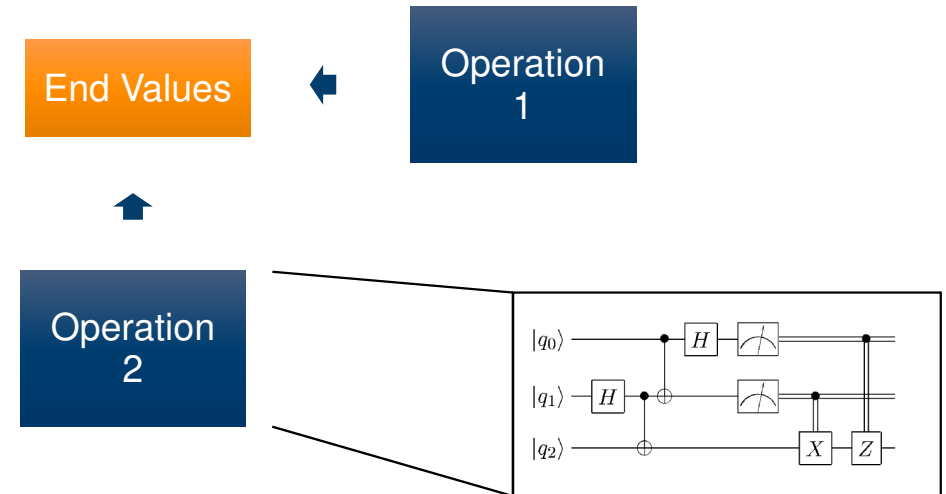
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Circuit Execution

Classical Computer



Quantum Computer



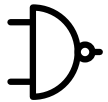
[1]

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Gates

Classical Computer

- Universal gate: NAND



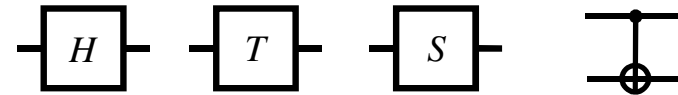
- Logical operation
- Truth tables

a	0	0	1	1
b	0	1	0	1
out	1	1	1	0

- Mostly one directional

Quantum Computer

- Universal gate set: {H,T,S,CNOT}



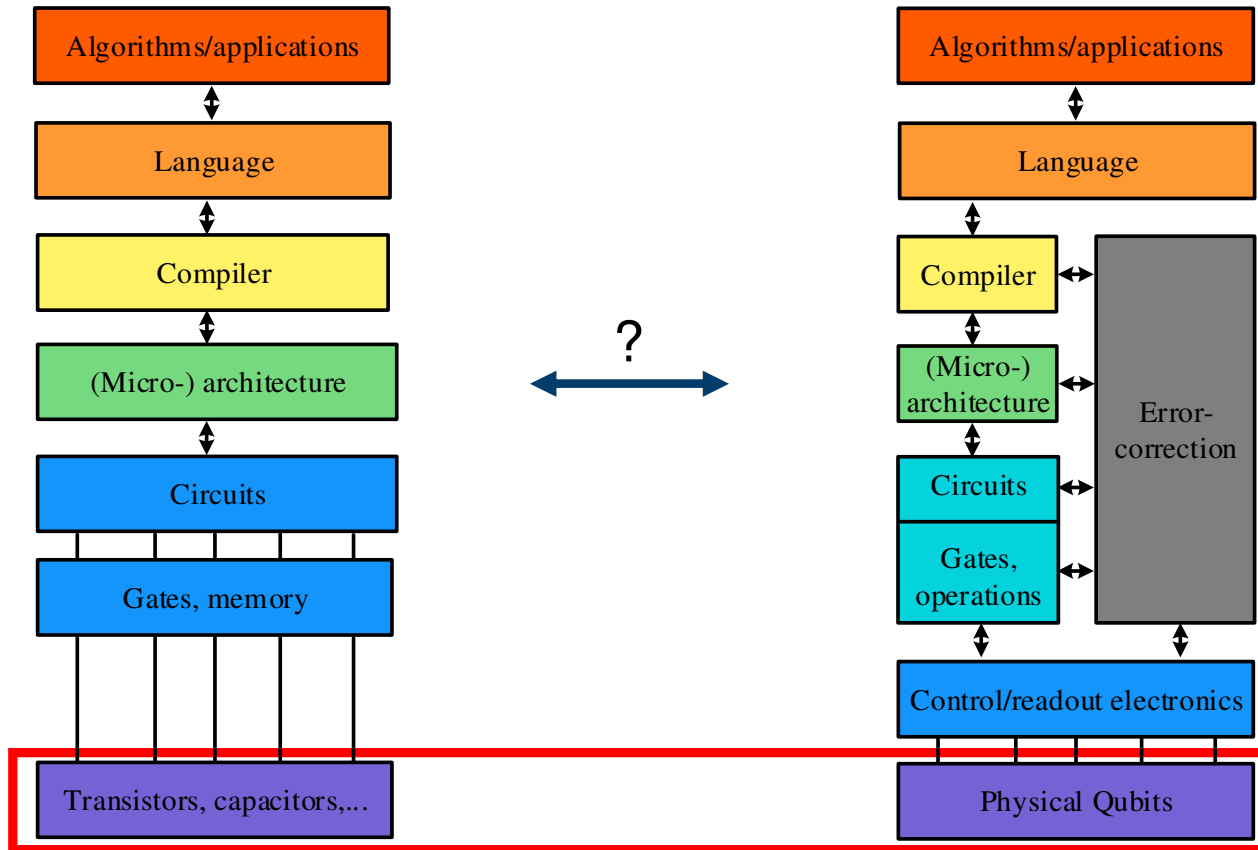
- Unitary operation
- Unitary matrix

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix} \quad \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

- Reversible

[1]

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BASIC QUANTUM INFORMATION

The qubit

Term „Qubit“:

Used as basic unit of quantum information

and

the technical realisation containing that information

„state of the qubit“

State of multiple qubits

Makes the state space bigger, e.g. 2qubits

qubit

State of a qubit: a two level quantum mechanical system

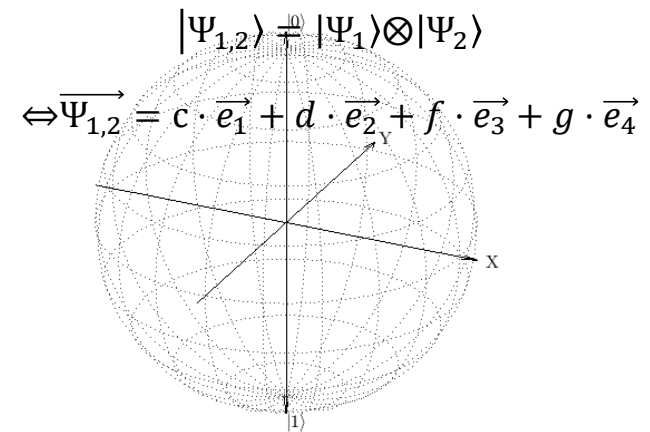
$$|\Psi\rangle = a |0\rangle + b |1\rangle$$

$$\Leftrightarrow \vec{\Psi} = a \cdot \vec{e}_1 + b \cdot \vec{e}_2$$

Superposition

$$|a|^2 + |b|^2 = 1 \text{ with } a, b \in \mathbb{C}$$

State readout: 0 or 1 with $|a|^2$ and $|b|^2$ probability



BASIC QUANTUM INFORMATION

Changing states and doing gates

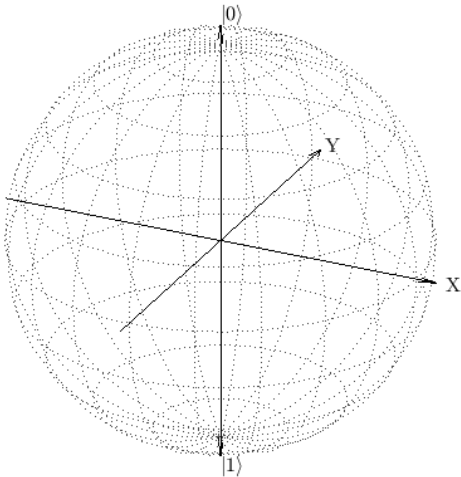
Single qubit gate = rotations in Bloch sphere

→ Multiplication with unitary matrix

→ Rotation matrix e.g.:

$$|\Psi\rangle = U \cdot |\Psi_0\rangle$$

$$R_x(\pi) = \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix}$$



Multiple qubit gate:

→ Multiplication with unitary matrix

$$\text{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

→ Example:

$$|\Psi_{1,2}\rangle = \text{CNOT} \cdot |\Psi_{0(1,2)}\rangle$$

→ But:

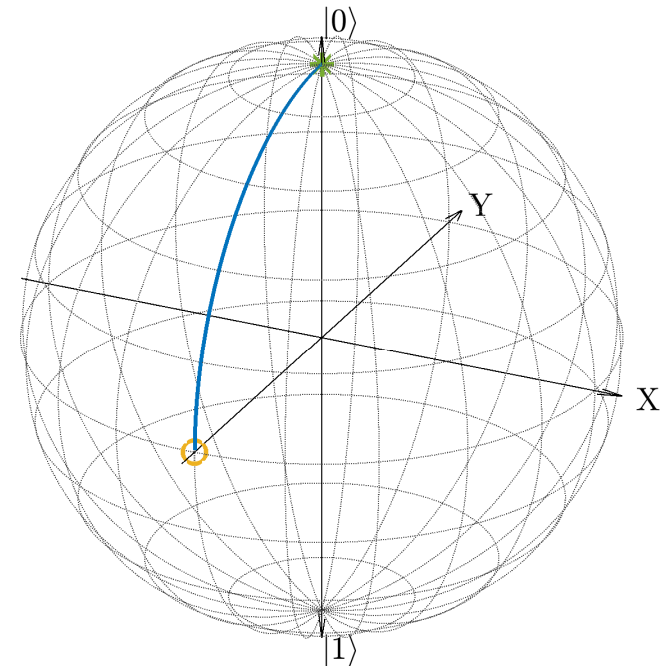
$$|\Psi_{1,2}\rangle \neq |\Psi_1\rangle \otimes |\Psi_2\rangle \quad \text{Entanglement}$$

BASIC QUANTUM INFORMATION

Exercise 1

How to get from $\star$ to $\circ$?

$$\begin{aligned} R_x(\theta) &\equiv e^{-i\frac{\theta}{2}X} = \cos\frac{\theta}{2}I - i\sin\frac{\theta}{2}X = \begin{bmatrix} \cos\frac{\theta}{2} & -i\sin\frac{\theta}{2} \\ -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix} \\ R_y(\theta) &\equiv e^{-i\frac{\theta}{2}Y} = \cos\frac{\theta}{2}I - i\sin\frac{\theta}{2}Y = \begin{bmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix} \\ R_z(\theta) &\equiv e^{-i\frac{\theta}{2}Z} = \cos\frac{\theta}{2}I - i\sin\frac{\theta}{2}Z = \begin{bmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{bmatrix} \end{aligned}$$



BASIC QUANTUM INFORMATION

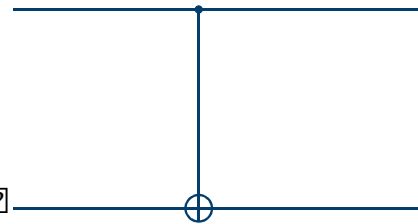
Exercise 2

What quantum circuit is this and what is the output?

$$|\Psi_{0,1}\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$|\Psi_{0,2}\rangle = 1 \cdot |0\rangle + 0 \cdot |1\rangle$$

input



$$|\Psi_{1,2}\rangle = ?$$

output

Help:

$$|A\rangle \otimes |B\rangle = \begin{bmatrix} A_{11}B & A_{12}B \\ A_{21}B & A_{22}B \end{bmatrix}$$

BASIC QUANTUM INFORMATION

Quality of gates

How do we determine the gate quality?

→ Calculate the fidelity (0,1):

$$F = \frac{1}{n^2} |\text{Tr}[U^{\text{T}}_{\text{ideal}} \cdot U]|^2$$

→ Benchmark set by error correction (infidelity):

$$1 - F = 10^{-2} - 10^{-3}$$

(current research^best between $10^{-1} - 10^{-2}$)

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Qubit implementations

Semiconductor Qubits

Trapped Ions

Superconducting Qubits



BASIC QUANTUM COMPUTING

GaAs qubit

Build:

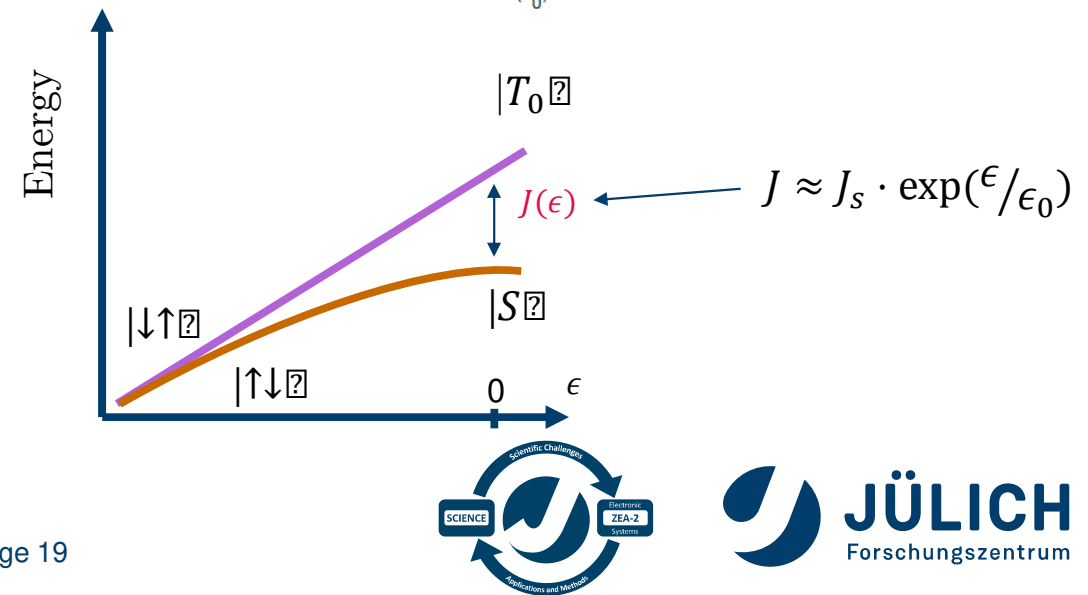
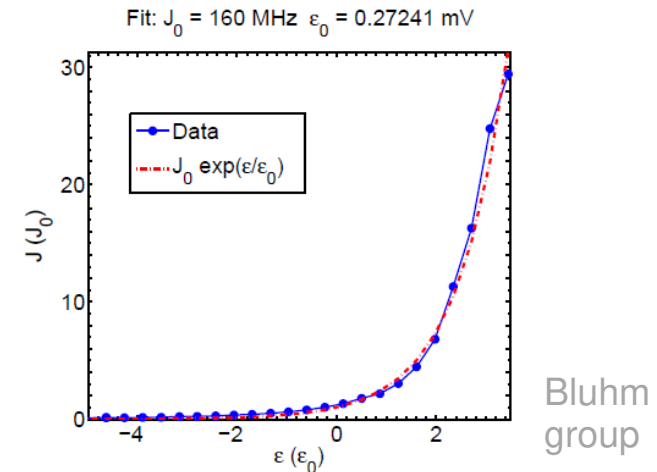
- 2D electron gas through hetero structure
- 2D confinement with topological electrodes
- Capture electrons in resulting double quantum dot

State:

- Use spin of electrons to encode state of qubit

$$|0\rangle = |T_0\rangle, |1\rangle = |S\rangle$$

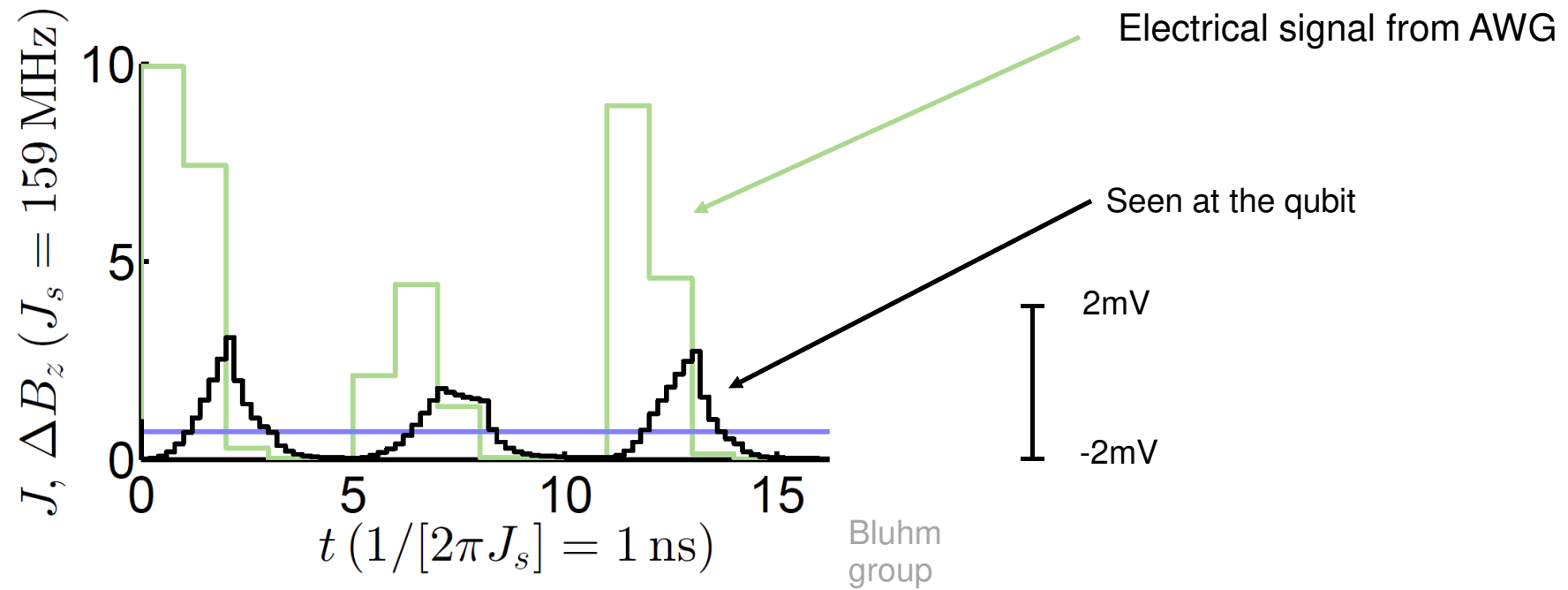
- State = energy state
- Manipulation through ϵ voltage signal



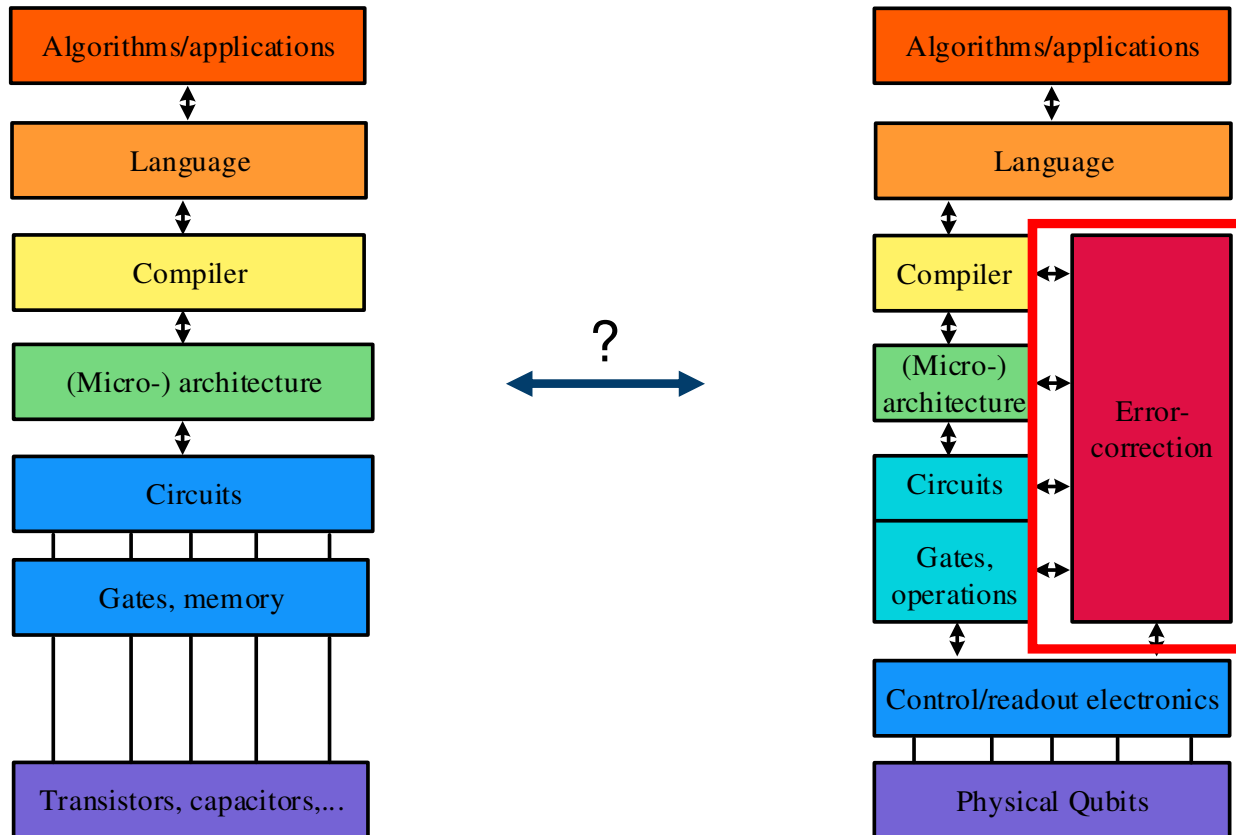
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Operating a GaAs qubit

Use rectangular sequences:



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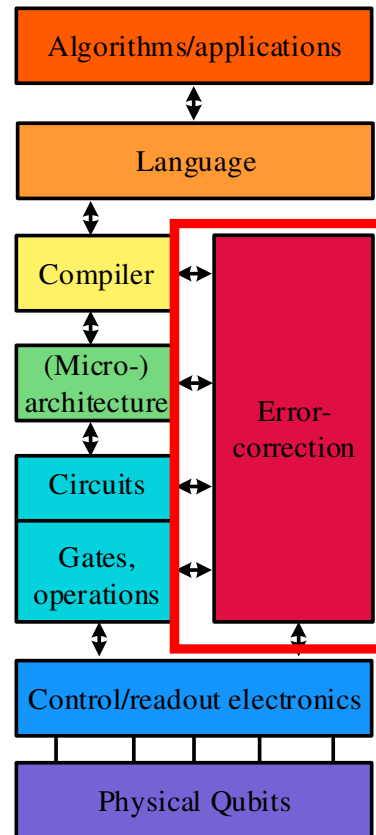
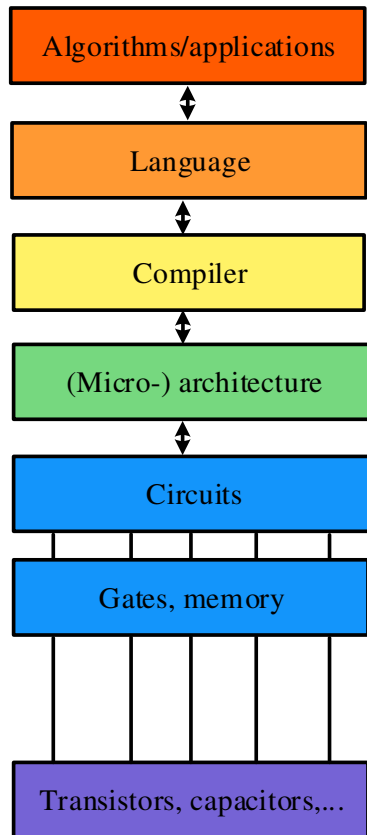
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Error Correction

- Short storage (coherence) time
 - $\mu\text{s} - \text{s}$
- Unreliable operation
 - Gate error rate: MOSFET 10^{-16} vs Qubit $10^{-1} \sim 10^{-2}$
- Limitations for quantum error correction
 - No copying
 - Destructive detection
 - Continuous error



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Number of qubits for
2048 bit Shor's algorithm
(prime factorization)

>5 billion

1.3 billion

100.000

[1] Similarly taught in EE4575 and AP3421 at Delft University of Technology

